STAT 2290 Homework 6

Problem 1.
In a recent National Survey of Drug Use and Health, 2312 of 5914 randomly selected full-time U.S. college students were classified as binge drinkers.
(a) Calculate and interpret a 99% confidence interval for the population proportion p that are binge drinkers.
(b) A newspaper article claims that 45% of full-time U.S. college students are binge drinkers. Use your result from part (a) to comment on this claim.

Answer (a):

Check conditions: Before doing any calculation for sample proportions, check 10% condition and Large Counts condition hold. If these hold, continue. If not, say which condition is violated and say we cannot proceed:

- **Sample size:** $n = 5914$.
- **Proportion:** $\hat{p} = \frac{2312}{5914} \approx 0.39$ of respondents answered "Yes" to our question of "Are you a binge drinker?".
- **10% Condition:** $10n = 10 \cdot 5914 = 59140$. There are surely more than this many full-time U.S. college students so this condition is met.
- **Large Counts:** $n\hat{p} = 5914 \cdot \frac{2312}{5914} = 2312 \geq 10$ and $n\hat{(1-p)} = 5914 \cdot (1 - \frac{2312}{5914}) = 5914 \cdot \frac{3602}{5914} = 3602 \geq 10$ so the Large Counts Condition is met.

Calculate: We are allowed to calculate the confidence interval.

- $c = 99\%$ so the corresponding critical value is $z_c = 2.575$.
- the margin of error is

$$E = z_c \cdot \sqrt{\frac{p(1-p)}{n}} = z_c \cdot \sqrt{\frac{0.39(1-0.39)}{5914}} = 0.016$$

Confidence interval:
The 99% confidence interval is

$$39 - 1.6 = 37.4\% < p < 39 + 1.6 = 40.6\%$$

Interpretation: So we are 99% confident that the true proportion of full-time U.S. college students who are binge drinkers is between $39 - 1.6 = 37.4\%$ and $39 + 1.6 = 40.6\%$.

Answer (b):
(a) shows that out of every 100 surveys of size $n = 5914$, we expect only 1 of them would result in a sample proportion that lies outside our interval. From our data, the 45% claim is unlikely to be true.

Problem 2.
PTC is a substance that has a strong bitter taste for some people and is tasteless
for others. The ability to taste PTC is inherited. About 75% of Italians can taste PTC, for example. You want to estimate the proportion of Americans who have at least one Italian grandparent and who can taste PTC.
(a) How large a sample must you test to estimate the proportion of PTC tasters within 0.04 with 90% confidence? Answer this question using the 75% estimate as the guessed value for $\hat{p}$.
(b) Answer the question in part (a) again, but this time use the conservative guess $\hat{p}$ = 0.5. By how much do the two sample sizes differ?

Answer: This is a minimal sample size for sample proportion question.

(a) We are given :

- $\hat{p} = 0.75$,
- $c = 90\%$ so $z_c = 1.645$

We are asked to make the margin of error less than 0.04 so:

$$0.04 \geq E = z_c \cdot \sqrt{\frac{p(1-p)}{n}} = 1.645 \cdot \sqrt{\frac{0.75(1-0.75)}{n}}.$$

Solving for $\sqrt{n}$ gives

$$\sqrt{n} \geq 1.645\sqrt{0.75 \cdot 0.25}/0.04 = 17.8076473653$$

So $n \geq (17.8076473653)^2 = 317.1123046869$

So we need a minimal sample size of $n = 318$.

(b) We are given :

- $\hat{p} = 0.5$,
- $c = 90\%$ so $z_c = 1.645$

Again

$$0.04 \geq E = z_c \cdot \sqrt{\frac{p(1-p)}{n}} = 1.645 \cdot \sqrt{\frac{0.5(1-0.5)}{n}}.$$

Solving for $\sqrt{n}$ gives

$$\sqrt{n} \geq 1.645\sqrt{0.5 \cdot 0.5}/0.04 = 20.56$$

So $n \geq (20.56)^2 = 422.81$

So we need a minimal sample size of $n = 423$.

The difference between the two sample sizes is $423 - 318 = 105$.

Problem 3.
A group of researchers calculates the mean quantity of sodium (in milligrams) in selected branded cereals consumed by people in each serving. To do so, the group takes a random sample of 30 branded cereals and obtain the quantity (in milligrams) below.

```
130 15 260 140 200 180 125 210 200 210 220 290 210 140 180
280 290 90 180 140 80 220 140 190 125 200 0 160 240 135
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From past studies, the research council assumes that σ is 70.7 milligrams. Construct a 99% confidence intervals for the population mean.

Answer:

Check conditions:

- 10% Condition: $10n = 10$
- Large sample size/normality: $n = 30 \geq 30$ so this condition is met.

So we can proceed to calculations.

Calculate:

- The sample mean is $\bar{x} = \frac{5180}{30} = 172.67$.
- $c = 99\%$ so the corresponding critical value is $z_c = 2.575$.
- So the margin of error is

$$E = z_c \frac{\sigma}{\sqrt{n}} = 2.575 \cdot \frac{70.7}{\sqrt{30}} = 33.24$$

So the confidence interval is

$$172.67 - 33.24 < \mu < 172.67 + 33.24$$

Problem 4.
A soccer ball manufacturer wants to estimate the mean circumference of soccer balls within 0.15 inch. Determine the minimum sample size required to construct a 99% confidence interval for the population mean. Assume the population standard deviation is 0.5 inch.

Answer:

- $c = 99\%$ so the corresponding critical value is $z_c = 2.575$.
- $\sigma = 0.5$
- So the margin of error is

$$0.15 \geq E = z_c \frac{\sigma}{\sqrt{n}} = 2.575 \cdot \frac{0.5}{\sqrt{n}}$$

  Solving for n gives

$$n \geq (2.575 \cdot 0.5/0.15)^2 = 73.7$$

  So the minimal sample size needed is 74.